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NONLINEAR ANALYSIS OF LAMINATED BIMODULAR COMPOSITE  
MATERIAL STRUCTURES. (U) OKLAHOMA UNIV NORMAN SCHOOL OF  
AEROSPACE MECHANICAL AND NUCLE. C W BERT ET AL.

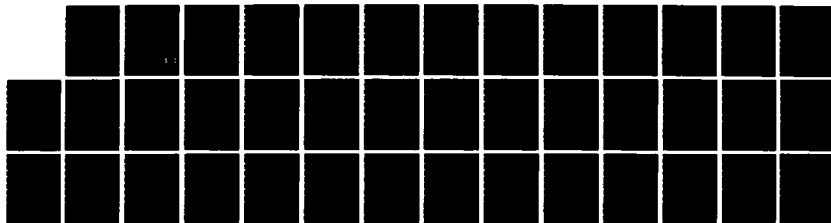
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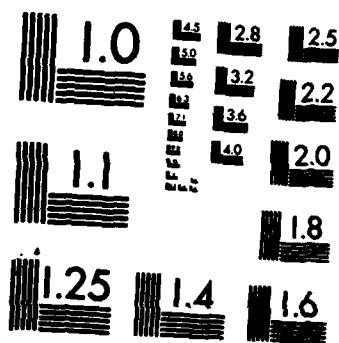
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NONLINEAR ANALYSIS OF LAMINATED, BIMODULAR,  
COMPOSITE MATERIAL STRUCTURES

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### Preface

The research summarized herein was conducted during the period 1978-83 under Contract N00014-78-C-0647 to the University of Oklahoma from the Mechanics Division of the Office of Naval Research (ONR). In the first two years, all of the research was conducted at the University of Oklahoma. In the last three years, half of the research was conducted under the direction of Professor Bert at the University of Oklahoma and the remainder was conducted under the direction of Professor Reddy at the Virginia Polytechnic Institute and State University under a subcontract from the University of Oklahoma.

The authors gratefully acknowledge the encouragement and support of Dr. Nicholas Perrone, Dr. Y. Rajapakse, and Dr. Nicholas L. Basdekas of ONR, who served as technical monitors of the program. The authors would like to thank Professor T. Kuppusamy of the Department of Civil Engineering at the Virginia Polytechnic Institute for his contributions to the three-dimensional finite-element work. Also, the authors would like to acknowledge the technical contributions to this program made by the thirteen graduate students, whose names are listed in the List of Project Personnel at the end of this report.

Finally, the skillful typing of Mrs. Rose Benda at OU and of Mrs. Vanessa McCoy at VPI is appreciated.

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### Abstract

This final report summarizes in a compact form the results of a five-year combined analytical, numerical, and experimental research program on structures (plates, shells, and beams) constructed of fiber-reinforced, soft-matrix composite materials which have significantly different stress-strain behavior depending upon whether the fibers are stretched or compressed. Emphasis is placed on advances made in these six areas:

1. Material modeling of bimodular composite materials (BCMs)
2. Linear analyses of structures made of BCMs
3. Geometrically nonlinear analyses of BCM structures
4. Experimental investigations of BCMs and structures
5. Analyses of structures made of nonlinear composite materials
6. New structural analyses

The research program was reported in a series of thirty-seven technical reports (including the present one) and sixty-eight conference papers and journal articles, all of which are listed herein.

Key Words

Analysis, classical  
Analysis, finite-element  
Analysis, transfer-matrix  
Analysis, nonlinear

Beams, thick

Construction, sandwich

Elasticity, three-dimensional

Loading, dynamic  
Loading, static mechanical  
Loading, thermal

Materials, bimodular  
Materials, composite  
Materials, fiber-reinforced composite  
Materials, multimodular  
Materials, nonlinear  
Materials, plastic-range  
Models, material

Nonlinearity, geometric  
Nonlinearity, material

Plasticity  
Plates, laminated

Rings, thick

Shells, laminated  
Shells, thick  
Shells, thin  
Structures, laminated  
Structures, sandwich

Theory, plate  
Theory, shell  
Thermoelasticity  
Transverse shear deformation

Vibration, forced  
Vibration, free  
Vibration, linear  
Vibration, nonlinear  
Vibration, transient

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### Introduction

Before summarizing the research conducted under the present contract, it is desirable to briefly describe the state of the art prior to initiation of this work.

The concept of a material having different behavior in compression than in tension was known at least as far back as 1864, as mentioned by Saint-Venant<sup>1</sup>. The introduction of a bilinear approximation for such a material was introduced in 1941 by Timoshenko<sup>2</sup>. In both of these works, only uniaxial bending (beam action) was considered. The first multidimensional model for these materials and the coining of the word "bimodulus" or "bimodular" for the bilinear approximation is generally attributed to Ambartsumyan<sup>3</sup> in 1965. This model was for isotropic bimodular materials only, but was later extended to orthotropic bimodular materials<sup>4</sup>. Several other multidimensional bimodular material models were proposed, but none were very appropriate for materials with stiff unidirectional fibers and soft matrices. Examples of these fiber-governed materials include soft biological materials and cord-rubber, such as used in automotive tires, V-belts, and air-cushion-vehicle skirts; see Table 1. In 1977,

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<sup>1</sup> Saint-Venant, B., Notes to Navier's Résumé des leçons de la résistance des corps solides, 3rd ed., Paris, 1864, p. 175.

<sup>2</sup> Timoshenko, S., Strength of Materials, Part II, Advanced Theory and Problems, 2nd ed., Van Nostrand, Princeton, NJ, 1941, pp. 368-369.

<sup>3</sup> Ambartsumyan, S.A., "The Axisymmetric Problem of Circular Cylindrical Shell Made of Materials with Different Stiffness in Tension and Compression," Izvestiya Akademii Nauk SSSR, Mekhanika, 1965, No. 4, pp. 77-85; English translation, NTIS Report FTD-HT-23-1055-67, National Technical Information Service, Springfield, VA, 1967.

<sup>4</sup> Ambartsumyan, S.A., "The Basic Equations and Relations of the Different-Modulus Theory of Elasticity of an Anisotropic Body," Mechanics of Solids, Vol. 4, No. 3, 1969, pp. 48-56.



Bert<sup>5</sup> introduced a model for fiber-governed materials and successfully applied it to experimental data reported by Patel et al.<sup>6</sup>

Table 1. Some Examples of Materials Having Different Elastic Behavior in Tension and Compression

<u>Investigator</u>	<u>Material</u>	<u>Ratio of Young's Moduli <math>E_C/E_T</math></u>
Zolutukhina & Lepetov <sup>7</sup>	Various fabric/rubber	0.07 to 0.50
Patel et al. <sup>6</sup>	Polyester cord/rubber	0.017
	Aramid cord/rubber	0.0034
Ducheyne et al. <sup>8</sup>	Sintered, porous stainless steel	0.1
Pearsall & Roberts <sup>9</sup>	Myometrium (uterine muscle)	0.2

Regarding the experimental characterization of the mechanical behavior of materials with drastically different behavior in tension and compression, only a very limited amount of information had been published. All of the work known to the present investigators was limited to either stress-strain curves in one direction (such as in Refs. 7-9) or reporting

<sup>5</sup> Bert, C.W., "Models for Fibrous Composites with Different Properties in Tension and Compression," ASME Journal of Engineering Materials and Technology, Vol. 99H, No. 3, 1977, pp. 344-349.

<sup>6</sup> Patel, H.P., Turner, J.L., and Walter, J.D., "Radial Tire Cord-Rubber Composites," Rubber Chemistry and Technology, Vol. 49, 1976, pp. 1095-1110.

<sup>7</sup> Zolutukhina, L.I. and Lepetov, V.A., "The Elastic Modulus of Flat Rubber-Fabric Constructions in Elongation and Compression," Soviet Rubber Technology, Vol. 27, No. 10, 1968, pp. 42-44.

<sup>8</sup> Ducheyne, P., Aeronouat, E., and de Meester, P., "The Mechanical Behaviour of Porous Austenitic Stainless Steel Fibre Structures," Journal of Materials Science, Vol. 13, 1978, pp. 2650-2658.

<sup>9</sup> Pearsall, G.W. and Roberts, V.L., "Passive Mechanical Properties of Uterine Muscle (Myometrium) Tested in Vitro," Journal of Biomechanics, Vol. 11, 1978, pp. 167-176.

of only the separate moduli in tension and compression (such as in Ref. 6) rather than complete stress-strain curves in various directions.

There had been extensive analytical investigations of structural elements (beams, plates, and shells) made of bimodular materials; however, most of these were limited to homogeneous isotropic bimodular materials. Notable exceptions include the works of Kotlyarskii and Karbasova<sup>10</sup> and Crawford<sup>11</sup> on laminate bending; Kamiya<sup>12</sup> on deflection of single-layer, orthotropic, circular cylindrical shells; and Jones<sup>13</sup> and Jones and Morgan<sup>14</sup> on buckling of laminated orthotropic circular cylindrical shells.

Finally in the area of finite-element modeling of bimodular structural elements, the present investigators know of no previous investigations.

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<sup>10</sup> Kotlyarskii, I.M. and Karbasova, I.M., "Determining the Forces Occurring in the Cross-Section of a Conveyor Belt when it is Being Bent Round a Drum," Soviet Rubber Technology, Vol. 27, No. 3, 1968, pp. 38-40.

<sup>11</sup> Crawford, R.F., "An Evaluation of Boron-Polymer Film Layer Composites for High-Performance Structures," NASA CR-1114, 1968.

<sup>12</sup> Kamiya, N., "Axisymmetric Deformation of Bimodulus Orthotropic Cylindrical Shell," Journal of the Engineering Mechanics Division, Proc. ASCE, Vol. 102, 1976, pp. 89-103.

<sup>13</sup> Jones, R.M., "Buckling of Stiffened Multilayered Circular Cylindrical Shells with Different Orthotropic Moduli in Tension and Compression," AIAA Journal, Vol. 9, 1971, pp. 917-923.

<sup>14</sup> Jones, R.M. and Morgan, H.S., "Buckling of Laminated Circular Cylindrical Shells with Different Moduli in Tension and Compression," Proceedings of the 1975 International Conference on Composite Materials (ICCM-I), Vol. 2, Geneva, Switzerland and Boston, MA, 1975, Metallurgical Society of AIME, New York, 1975, pp. 318-343.

### Summary of Research Progress

The research progress accomplished in the present project is most conveniently discussed in five major categories, each of which is covered in the ensuing subsections.

#### 1. Material Modeling of Composite Materials Having Difference Stress-Strain Behavior in Tension and Compression

In conjunction with selection of the best macroscopic model for fiber-governed bimodular materials, Bert (P-1 and TR-1)\* conducted a critical evaluation of numerous theories of this type.

A comprehensive micromechanics model for fiber-governed materials was also presented by Bert (P-8 and TR-1). This model qualitatively explained the mechanical behavior of cord-reinforced, soft-matrix materials: the increasing stiffness when the fibers are in tension ("tie-bar" action) and decreasing stiffness ("column-on-elastic-foundation" action) when the fibers are compressed. This model also predicted a bimodular effect on thermal-expansion behavior.

Another material model, based on the concept of continuum damage, involved extension of linear continuum damage theory to the nonlinear case (P-60 and TR-30).

#### 2. Linear Analyses of Structures Made of Bimodular Composite Materials

Apparently the first formulation and solution of a plate-bending problem for orthotropic bimodular material was due to Bert and Kincannon (P-3 and TR-4), who considered a clamped elliptic plate. They used

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\* In this and the ensuing sections, the notation P-1 refers to Paper No. 1 in the List of Conference Papers and Journal Articles, and TR-1 refers to Technical Report No. 1 in the List of Technical Reports.

Bert's fiber-governed symmetric material model<sup>5</sup> and mentioned the importance of the neutral-surface position associated with a change in sign of the total strain in the fiber direction. This is believed to be the first use of change in sign of total *strain* as the bimodularity criterion, as opposed to the change in sign of *stress* as was used earlier by Jones<sup>13</sup>. Since the analysis of structural elements, both thin and relatively thick, is based on the linearity of the distribution of the tangential displacements through the thickness, this alleviates the unstable convergence problems encountered by Jones.

Laminated bimodular plates were first analyzed by Reddy and Bert (P-9 and TR-2) by the finite-element method (FEM). They considered cross-ply rectangular plates with transverse shear deformation. Later closed-form solutions were presented for thin elliptic plates by Kincannon et al. (P-11 and TR-4) and for thin rectangular ones by Bert et al. (P-13 and TR-4). A series of papers by Bert and Reddy and their students presented both closed-form and finite-element results for the following problems of rectangular plates including transverse shear deformation: plate bending due to pressure loading (P-10 and TR-3; P-31 and TR-11), plate bending due to thermal loading (P-19 and TR-16), and plate vibration (P-23 and TR-15). Further FEM analyses of such plates were made by Reddy and Chao (P-15 and TR-7) and the results are summarized in Figs. 1 and 2. Transient analysis of such plates was carried out by Reddy (P-37 and TR-24). The project work on linear analysis of bimodular composite plates was summarized by Reddy and Bert (P-28, P-44, and TR-20).

Both closed-form and transfer-matrix analyses of the behavior of bimodular beams, including transverse shear deformation, were presented by Bert and Tran. They investigated both static (P-27 and TR-22) and

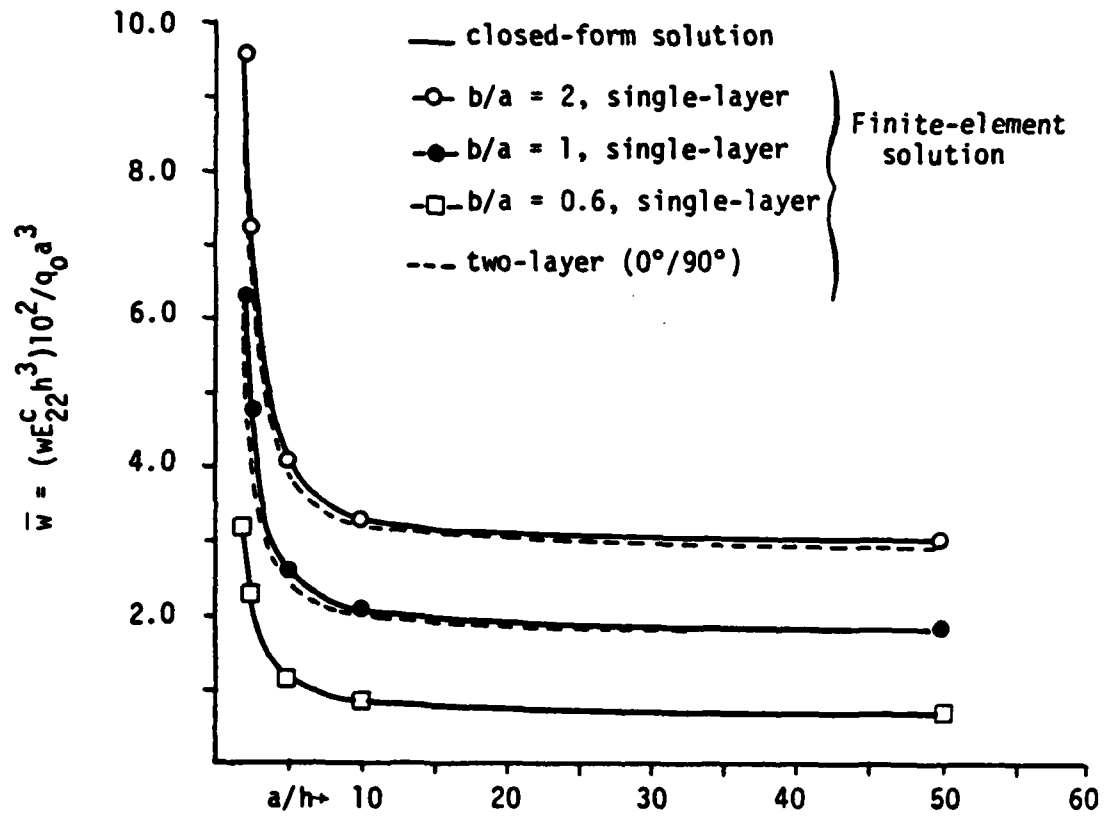


Figure 1 Effect of side-to-thickness ratio ( $a/h$ ) on the nondimensionalized deflection ( $w$ ) of single-layer ( $0^\circ$ ) and two-layer ( $0^\circ/90^\circ$ ) rectangular plates of aramid-rubber bimodular material (see P-55).

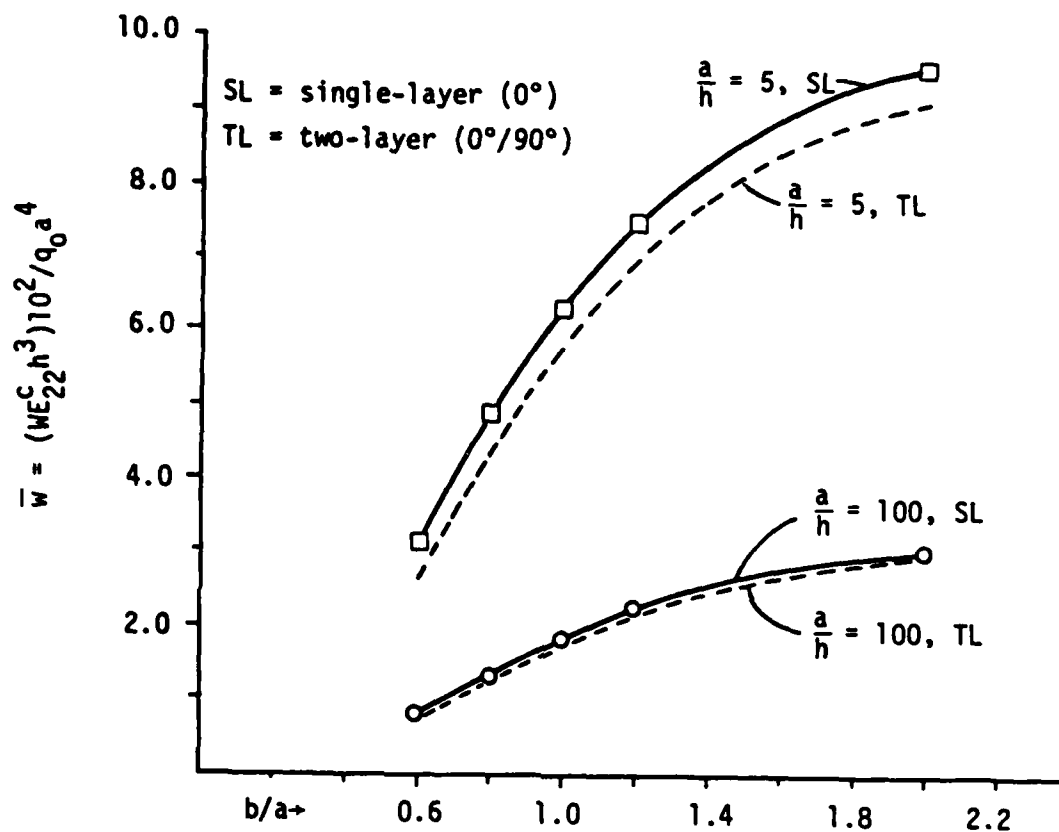


Figure 2 Effect of aspect ratio ( $b/a$ ) on the nondimensionalized deflection ( $w$ ) of single-layer (0°) and two-layer (0°/90°) rectangular plates of aramid-rubber bimodular material (see P-55).

transient (P-45 and TR-22) loadings. The agreement between the results obtained by the two methods was very good. Although the reduced computational effort of the transfer-matrix method was already well known, this work established its high accuracy as well. Further work on stacking-sequence effects on multilayered bimodular beams was presented by Bert and C.J. Rebello (P-56 and TR-26). Finally, a derivation of the effect of bimodularity on the Timoshenko shear-correction factor was presented by Bert and Gordaninejad (P-58 and TR-32).

The deflection of laminated bimodular shells was first analyzed in 1980 by Bert and V.S. Reddy, who presented closed-form solutions for thin cylindrical panels under pressure loading (P-17 and TR-12). Later, thick laminated bimodular shells were first analyzed by Hsu et al., who presented both closed-form and finite-element analyses for mechanical and thermal loadings (P-20, P-27, and TR-17). Vibrations of both thin and thick laminated bimodular cylindrical shells were investigated by Bert and Kumar (P-21 and TR-19).

Most of the project work on linear analysis of laminated bimodular composite material beams, plates, and shells, as well as contributions throughout the world, were summarized in an invited survey paper by Bert and Reddy (P-53 and TR-28) at a recent IUTAM (International Union of Theoretical and Applied Mechanics) symposium.

### 3. Geometrically Nonlinear Analyses of Structures Made of Bimodular Composite Materials

Due to the variation of the neutral-surface location with position on the middle surface of a bimodular-material structure undergoing large

deflections, the stiffnesses of such a structure change with position along the middle surface. Thus, the analysis of geometrically nonlinear problems is too complicated to permit closed-form solutions. This means that either an approximate analytical technique or a numerical technique is required. In a series of two papers, Reddy and Chao used the latter approach.

The first geometrically nonlinear analysis of a laminated bimodular structure was conducted by Reddy and Chao (P-55 and TR-25). They analyzed both large-deflection static behavior and free vibration of bimodular composite-material plates. Figure 3 contains plots of center deflection versus load for single- and two-layer cross-ply plates under uniform loading.

#### 4. Experimental Investigations of Bimodular Materials and Structures

Due to the limited experimental information available on the mechanical behavior of bimodular composite materials and structures constructed of them, it was deemed advisable to conduct some experiments along these lines.

The first problem to overcome in order to characterize the mechanical behavior was to find a suitable means of measuring strains in materials having such low moduli. Ordinary metallic foil gages are so stiff, relative to the material which it is desired to test, that measurements with such gages are more a measure of the mechanical behavior of the gages than the material on which they are mounted. It was decided to use liquid-metal strain gages, primarily due to their exceedingly low stiffness. Details of the manufacture, installation, calibration, and use of such gages were reported (P-40 and TR-23).

The next problems were how to load the material in compression without



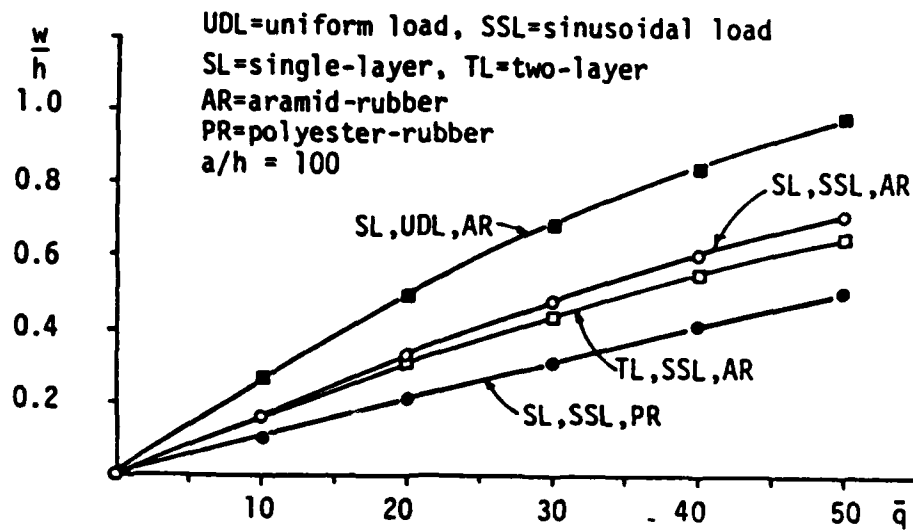


Figure 3a Load-deflection curves for thin square plates of bimodular materials ( $a/h = 100$ )

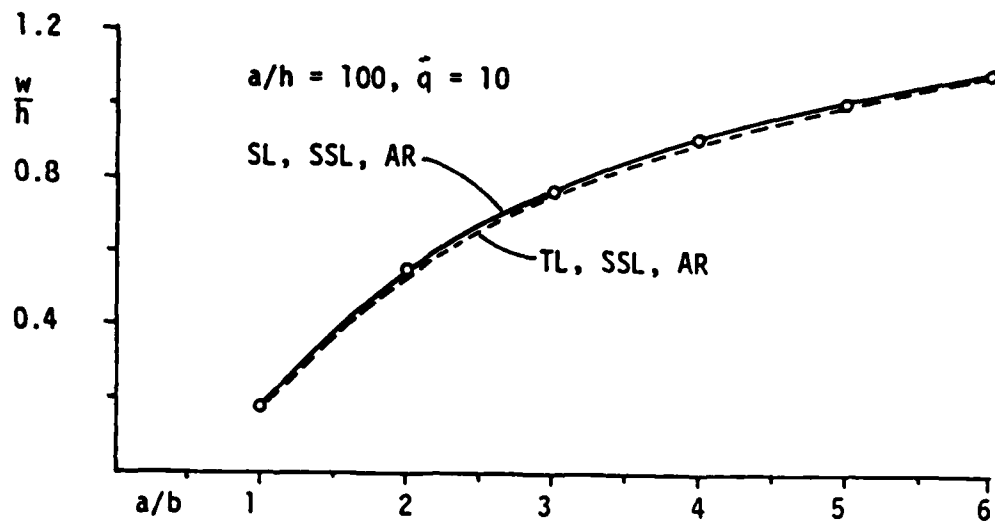


Figure 3b Effect of plate aspect ratio on the nonlinear deflection of aramid-rubber bimodular-material rectangular plates under sinusoidal loading (SSL) (SL=single-layer, TL=two-layer, AR=aramid-rubber, PR=polyester-rubber)

inducing buckling instability and how to obtain shear-moduli data. Both of these problems were overcome with the same solution: use of sandwich-beam specimens with facings of the bimodular material being investigated. In fact, it turned out that this specimen configuration offers a simple way of obtaining both tensile and compressive stress-strain curves simultaneously, as reported (P-46 and TR-23).

The overall experimental program, including the data-reduction procedures, was described in (P-36 and TR-23) and extensive experimental data were reported for aramid-cord/rubber, polyester-cord/rubber, and "wavy" steel-cord/rubber (P-38 and TR-23). It is believed that this work was the first complete experimental characterization of the mechanical behavior of tire-cord/rubber materials reported in the literature. It is important to emphasize that the material behavior is exactly the opposite of that of the usual metal-matrix and polymer-matrix composites: it is most non-linear  $0^\circ$  and  $90^\circ$  to the fiber direction and most linear at  $45^\circ$  (see Figs. 4, 5, and 6, taken from TR-23).

As an example of a structural element constructed of a bimodular composite material, a sandwich beam was selected and an experimental vibration investigation was conducted on it by Bert and C.A. Rebello (P-51, P-59, and TR-26). The agreement between the experimentally determined resonant frequencies and those predicted using the new shear-correction factor presented by Bert and Gordaninejad (P-58 and TR-32; see subsection 1 of this report) was excellent (see Fig. 7).

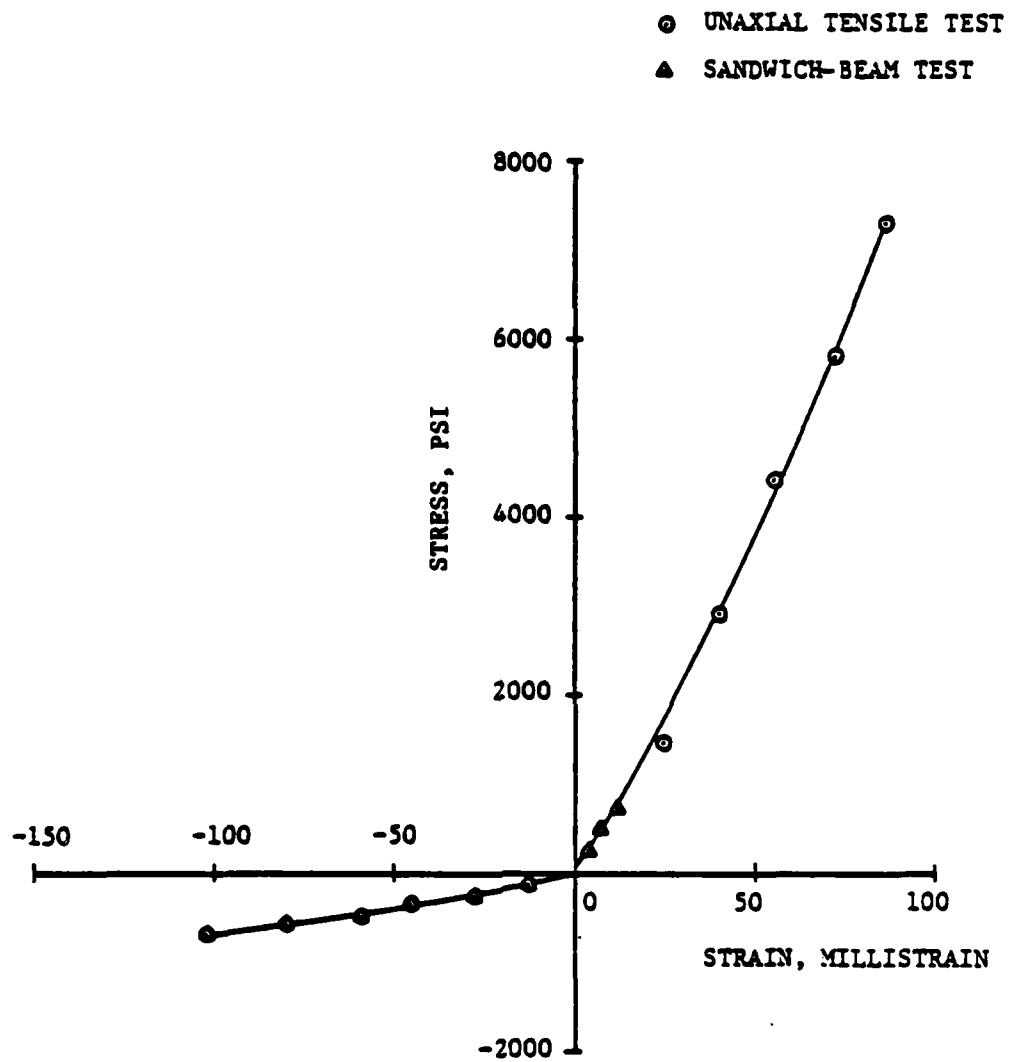


Fig. 4. Stress-strain curve for polyester-rubber in the cord direction (1 psi =  $6.895 \times 10^3$  pascals).

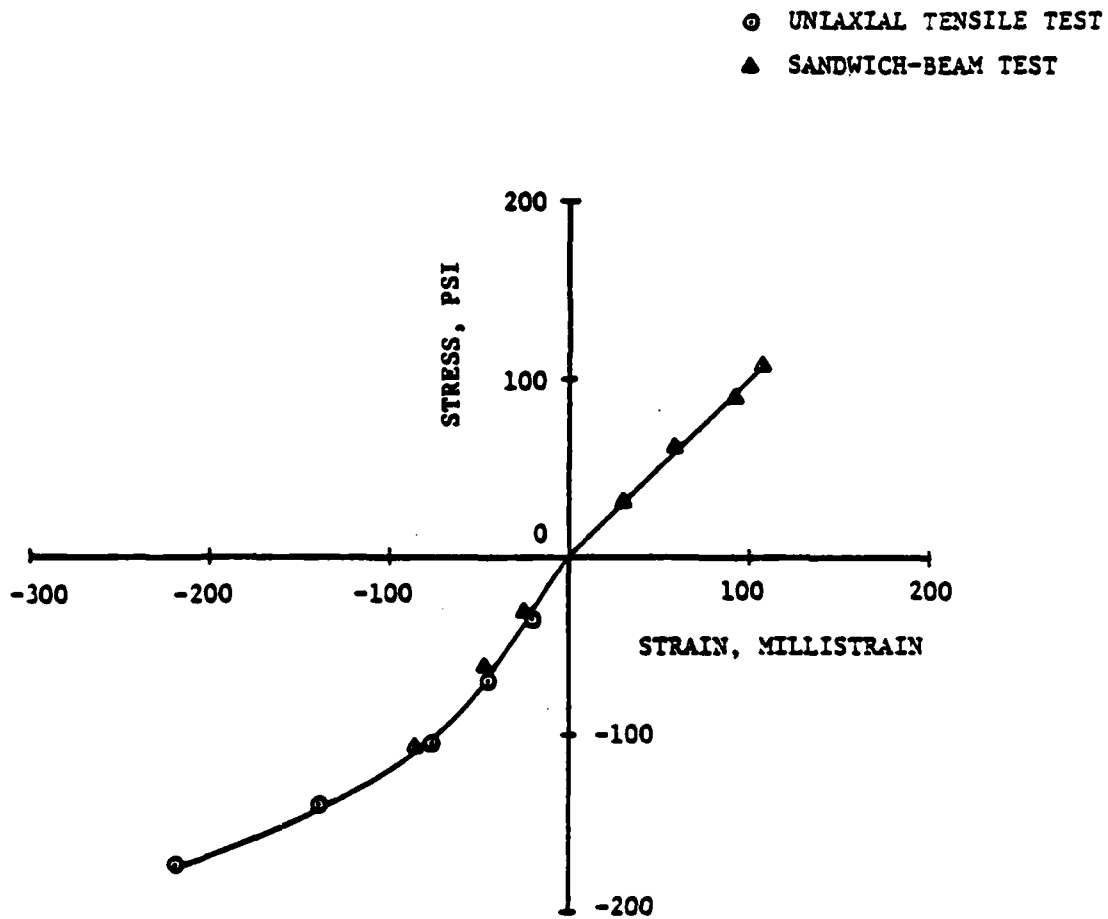


Fig. 5. Stress-strain curve for polyester-rubber in the transverse direction ( $1 \text{ psi} = 6.895 \times 10^3 \text{ pascals}$ ).

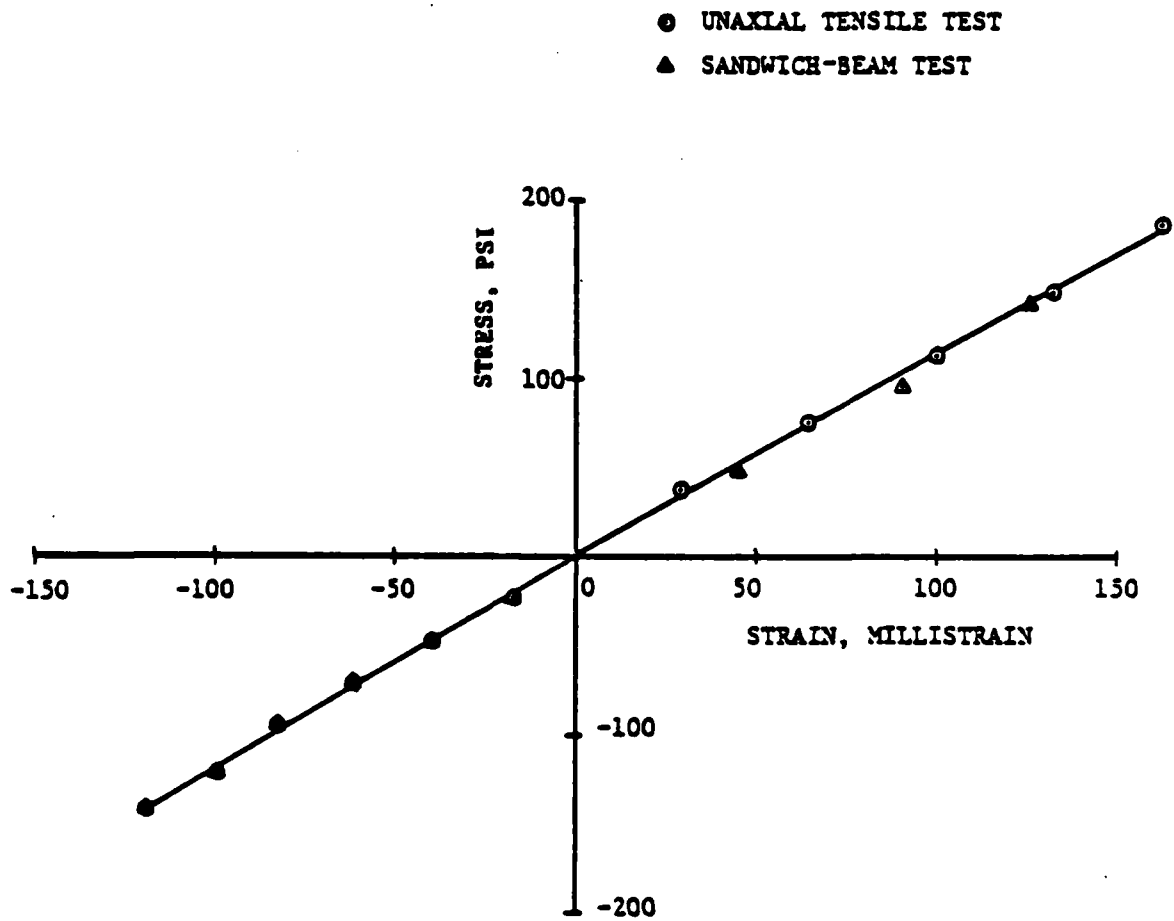


Fig. 6. Stress-strain curve for polyester-rubber at 45° to the cord direction (1 psi =  $6.895 \times 10^3$  pascals).

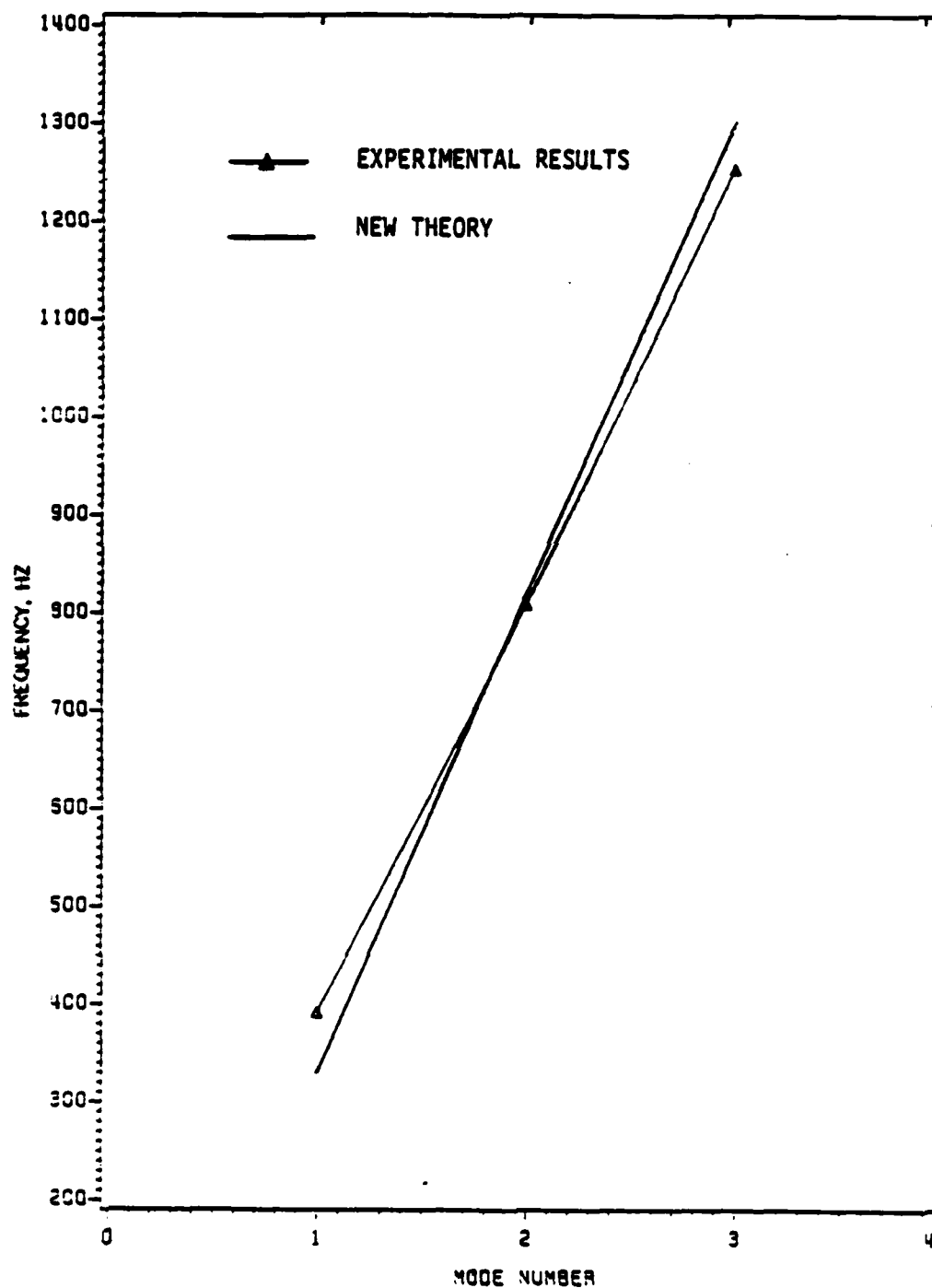


Fig. 7. Frequency vs. mode number for beam with 0° facings.

## 5. Analyses of Structures Made of Nonlinear Composite Materials

Three different kinds of investigations in this area were conducted in the project. The first was an analytical investigation by Bert (P-60 and TR-30) to predict the bending strength of a nonlinear material having different nonlinear stress-strain curves in tension and compression. The material was modeled using the continuum damage concept and the results agreed well with published data on concrete.

The second kind of investigation embodied the transfer-matrix analyses by Gordaninejad and Bert on the behavior of beams constructed of materials having different nonlinear stress-strain curves in tension and compression. The stress-strain curves were approximated by various numbers of piecewise linear segments curve-fitted to cord-rubber experimental data by a least-squares approach. Both static (P-39, P-54, P-64, and TR-30) and dynamic (P-61 and TR-30) analyses were conducted.

The third kind of investigation was a finite-element analysis of materially nonlinear, laminated composite plates by Kuppusamy, Nanda, and Reddy (P-66 and TR-34). Figure 8 contains the load-deflection curves obtained using various material models for three-layer cross-ply, simply supported, square plates under uniform load.

## 6. New Structural Analyses

In the first four years of the project, incidental to the development of the various finite-element analyses of bimodular-composite plates and shells as described in Sections 2, 3, and 5, Reddy made numerous investigations of plates and shells constructed of ordinary (not bimodular) materials. Notable contributions were the simplified analysis of nonlinear plate deflection (P-2 and TR-3) and plate free vibration (P-6 and TR-3) and the penalty-method plate-bending element (P-4, P-14, and TR-6). The

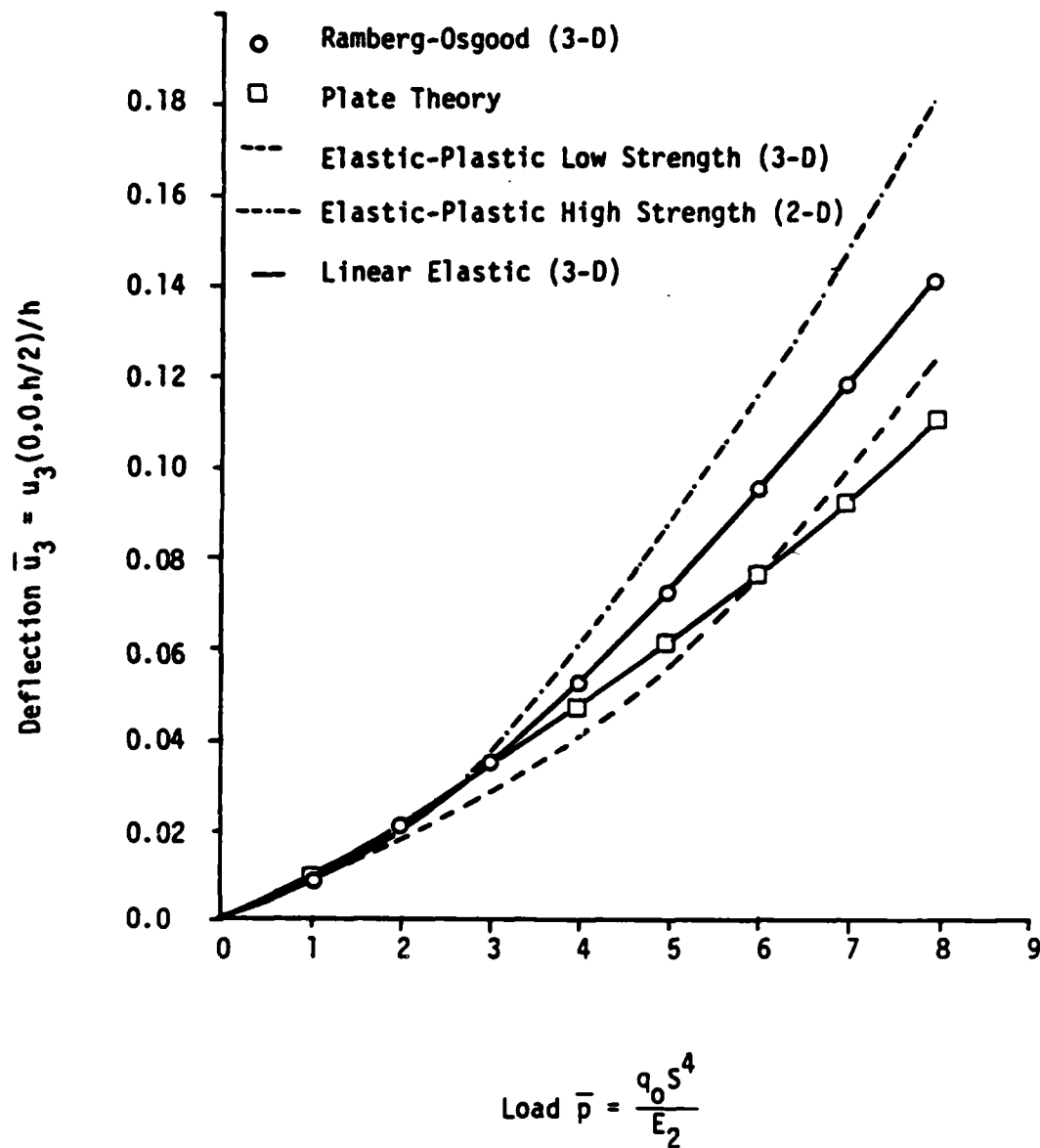


Figure 8 Load-Deflection Curve for Cross-Ply ( $0^\circ/90^\circ/0^\circ$ , Equal Thickness Layers) Square Plate Under Uniformly Distributed Load  $q_0$  ( $S = a/h = 10$ ).



effects of transverse shear deformation were studied (P-16 and TR-9; P-25 and TR-8). Finite-element analyses of geometrically nonlinear plates included: (P-30, P-32, P-34, and TR-14); (P-41 and TR-21); and (P-48, P-50, and TR-25).

In conjunction with the linear shell analyses discussed in Section 2, the Sanders theory of thin shells was extended to include transverse shear deformation (P-20 and TR-17, P-21 and TR-19). This is believed to be the first time such an extension was presented. The importance of this is that the Sanders theory has been generally recognized as an excellent thin shell theory; now it is available in a thick-shell version as well.

In the final year of the project, primary emphasis was placed on new structural analyses. Bert extended the Levinson plate theory<sup>15</sup>, which provides for a more realistic distribution of transverse shear strain than does Reissner-Mindlin shear deformable plate theory, to laminated plates and made comparisons with numerous other theories (P-62, P-68, and TR-35).

Also, Bert developed a new theory for the in-plane behavior of shear-deformable rings, including those laminated of composite materials (P-57, P-67, and TR-36). This theory is based on the same philosophy as the Levinson beam theory<sup>16</sup>. It provides more accuracy than Bresse-Timoshenko type shear deformable theory and yet has less complexity than an analysis based on the theory of elasticity.

Finally, a series of papers by Reddy and his associates was concerned

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<sup>15</sup> Levinson, M., "An Accurate, Simple Theory of the Statics and Dynamics of Elastic Plates," Mechanics Research Communications, Vol. 7, 1980, pp. 343-350.

<sup>16</sup> Levinson, M. "A New Rectangular Beam Theory," Journal of Sound and Vibration, Vol. 74, 1981, pp. 81-87.

with finite elements based on three-dimensional elasticity. It was applied to linear and nonlinear problems of laminated plates, both static (P-33, P-52, and TR-29) and dynamic (P-56 and TR-31), as well as to the geometrically nonlinear static case (P-36 and TR-31).

### Technological Significance and Naval Relevance

The first three years of the research involved the application of the fiber-governed bimodular model for soft-matrix composite material, previously introduced by the senior principal investigator, to laminated structures, including plates, shells, and beams undergoing static mechanical and thermal loadings and free, steady-state forced, and transient dynamic loadings.

The analytical methods used ranged from classical closed-form solutions for special cases, through transfer-matrix analyses for beams, through various two- and three-dimensional finite elements. This series of reports and resulting journal articles constitutes the largest body of research conducted by any single group on the subject of bimodular-composite-material structures.

Also, the complete experimental characterization of the mechanical behavior for three cord-reinforced, rubber-matrix unidirectional composite materials, all highly nonlinear in stress-strain response, was conducted. This work represents the only complete characterization for such nonlinear composite materials to date.

In the fourth year, the analytical work was extended from bilinear (bimodular) to piecewise linear (multimodular) and nonlinear materials. In the fifth year, emphasis was placed on improved theories of plates and rings and to three-dimensional finite-element analyses.

The results of this project are applicable, to either greater or lesser extent, to a very wide range of categories of systems of engineering interest, and naval interest in particular. These include:

1. Cord-reinforced rubber skirts for air-cushion vehicles
2. Cord-reinforced rubber tires and power-transmission belts

3. Braided-metal-reinforced rubber hose
4. Filament-wound shell structures, such as rocket-motor casings, deep-diving submersibles, piping, and other pressure vessels
5. Wire-reinforced solid-propellant rocket grains
6. Biological materials, especially soft tissues
7. Ablative materials, such as quartz-phenolic and carbon-carbon
8. Aircraft structural components (such as wing and fuselage panels), especially those reinforced by aramid fibers
9. Porous materials, including sintered metals, porous bio-implant materials, and geotechnical materials such as soil and rock
10. Concrete and masonry structures undergoing bending
11. Metal-matrix-composite structural elements, such as turbine blades

List of Technical Reports

1. Bert, C.W., "Mathematical Modeling and Micromechanics of Fiber-Reinforced Bimodulus Composite Materials," Technical Report No. 1 (OU-AMNE-79-1), June 1979.
2. Reddy, J.N. and Bert, C.W., "Analyses of Plates Constructed of Fiber-Reinforced Bimodulus Materials," Technical Report No. 2 (OU-AMNE-79-8), June 1979.
3. Reddy, J.N., "Finite-Element Analyses of Laminated Composite-Material Plates," Technical Report No. 3 (OU-AMNE-79-9), June 1979.
4. Bert, C.W., "Analysis of Laminated Bimodulus Composite-Material Plates," Technical Report No. 4A (OU-AMNE-79-10A), revised version, August 1979.
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6. Reddy, J.N., "A Penalty Plate-Bending Element for the Analysis of Laminated Anisotropic Composite Plates," Technical Report No. 6 (OU-AMNE-79-14), August 1979.
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List of Conference Papers and Journal Articles

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(b) Accepted

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67. Bert, C.W., "A New Theory for Shear-Deformable Rings," (based on Tech. Report No. 36), Israel Journal of Technology, to appear.
68. Bert, C.W., "A Critical Evaluation of New Plate Theories Applied to Laminated Composites," (based on Tech. Report No. 35), Composite Structures, to appear.

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C.L. Ko, 1982-83

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This final report summarizes in a compact form the results of a five-year combined analytical, numerical, and experimental research program on structures (plates, shells, and beams) constructed of fiber-reinforced, soft-matrix com- posite materials which have significantly different stress-strain behavior depending upon whether the fibers are stretched or compressed. Emphasis is placed on advances made in these six areas: (1) material modeling of bimodular composite materials (BCMs); (2) linear analyses of structures made of BCMS; (over)		

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19. Key Words - Cont'd

material models, geometric nonlinearity, material nonlinearity, plasticity, laminated plates, thick rings, laminated shells, thick shells, thin shells, laminated structures, sandwich structures, plate theory, shell theory, thermoelasticity, transverse shear deformation, forced vibration, free vibration, linear vibration, nonlinear vibration, transient vibration.

20. Abstract - Cont'd

(3) geometrically nonlinear analyses of BCM structures; (4) experimental investigations of BCMs and structures; (5) analyses of structures made of nonlinear composite materials, and (6) new structural analyses.

The research program was reported in a series of thirty-seven technical reports (including the present one) and sixty-eight conference papers and journal articles, all of which are listed herein.

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